

# Higgs Mass Bounds Separate Models of Electroweak Symmetry Breaking

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## ABSTRACT

Vacuum stability implies a lower limit on the mass of the higgs boson in the Standard Model (SM). In contrast, an upper limit on the lightest higgs mass can be calculated in supersymmetric (susy) models. The main uncertainty in each limit is the value of the top mass, which may now be fixed by the recent CDF result. We study the possibility that these bounds do not overlap, and find that

(i) a mass gap emerges at  $m_t \sim 160$  GeV between the SM and the Minimal Susy Standard Model (MSSM); and between the SM and the Minimal plus Singlet Susy Model [(M+1)SSM] if the independent scalar self-coupling of the latter is perturbatively small or if the  $\tan \beta$  parameter is large; this gap widens with increasing  $m_t$ ;

(ii) there is no overlap between the SM and the MSSM bounds at even smaller values of  $m_t$  for the  $\tan \beta$  value ( $\sim 1-2$ ) preferred in Supersymmetric Grand Unified Theories.

Thus, if the new top mass measurement remains valid, a measurement of the first higgs mass will serve to exclude either the SM or MSSM/(M+1)SSM higgs sectors. In addition, we discuss the upper bound on the lightest higgs mass in susy models with an extended higgs sector, and in models with a strongly interacting higgs sector. Finally, we comment on the discovery potential for the lightest higgses in these models.

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# 1 Introduction

The simplest and best motivated possibilities for the electroweak symmetry breaking sector are the single higgs doublet of the minimal Standard Model SM, and the two higgs doublet sector of the Minimal Supersymmetric Standard Model (MSSM). Experimentally, very little is known about the higgs sector of the electroweak model. However, theoretically, quite a lot of higgs physics has been calculated. The electroweak symmetry-breaking scale is known: the vacuum expectation value (vev) of the complex higgs field  $\Phi$  is  $\langle 0|\Phi|0 \rangle = v_{SM}/\sqrt{2} = 175$  GeV. This value is remarkably close to the probable top quark mass of  $174 \pm 10^{+13}_{-12}$  GeV (very consistent with the SM prediction of  $m_t = 164 \pm 25$  GeV inferred from precision electroweak data [2]) announced recently by the CDF collaboration at Fermilab [1]. Higgs mass bounds have been calculated, including loop corrections. One aspect of the mass bounds [3] which we quantify in this paper is the following: inputting the CDF value for the top mass into quantum loop corrections for the symmetry-breaking higgs sector leads to mutually exclusive, reliable bounds on the SM higgs mass and on the lightest MSSM higgs mass. From this we infer that *if the CDF value for  $m_t$  is verified, then the first higgs mass measurement will rule out one of the two main contenders (SM vs. MSSM) for the electroweak theory, independent of any other measurement.*

There is another point to be made here. It is known that the Feynman rules connecting the lightest higgs in the MSSM to ordinary matter become, in the limit where the “other” higgs masses (these are  $m_A, m_H$ , and  $m_{H^\pm}$ , defined in section 3) are taken to infinity, exactly the SM Feynman rules [4]. When the masses are taken large compared to  $M_Z$ , of the order of a TeV, for example, the lightest MSSM higgs behaves very much like the SM higgs in its production channels and decay modes [5]; the only difference, a vestige of the underlying supersymmetry, is that the constrained higgs self coupling requires the MSSM higgs to be light, whereas SM vacuum stability requires the SM higgs to be heavy. Thus, *there may be no discernible difference between the lightest MSSM higgs and the SM higgs, except for their allowed mass values.* We demonstrate these allowed mass values in our Figures 1 and 2. Furthermore, the mass of the lightest MSSM higgs rises toward its upper bound as the “other” higgs masses are increased <sup>1</sup>. Thus, for masses in the region where the SM lower bound and the MSSM upper bound overlap, the SM higgs and the lightest MSSM higgs may not be distinguishable by branching ratio or width measurements. Only if the two bounds are separated by a gap is this ambiguity avoided.

In the SM and even in supersymmetric (susy) models the main uncertainty in radiative corrections is the value of the top mass. If the CDF announcement is confirmed, this main uncertainty is eliminated. *The radiatively corrected observable most sensitive to the value of the top mass is the mass of the lightest higgs particle in susy models* [6]: for large top mass, the top and scalar-top ( $\tilde{t}$ ) loops dominate all other loop corrections, and *the light higgs mass-squared grows as  $m_t^4 \ln(m_{\tilde{t}}/m_t)$ .* <sup>2</sup> We quantify this large correction in section 3.

<sup>1</sup>The saturation of the MSSM upper bound with increasing “other” higgs masses is well known in tree-level relations (the bound  $m_h \leq M_Z |\cos(2\beta)|$  approaches an equality as higgs masses increase) [4]. The MSSM upper bound still saturates with increasing “other” higgs masses even when one-loop corrections are included.

<sup>2</sup>It is not hard to understand this fourth power dependence; the contribution of the top loop to the SM higgs self energy also scales as  $m_t^4$ . However, in the SM the higgs mass is a free parameter at tree-level, and

In addition to contrasting the MSSM with the SM, we also consider in section 4 supersymmetric models with a non-standard Higgs sector, in particular the Minimal-plus-Singlet Susy Standard Model [(M+1)SSM] containing an additional SU(2) singlet, and a gauged non-linear sigma model. A discussion of supersymmetric grand unified theories (susy GUTs) is put forth in section 5; susy GUTs impose additional constraints on the low energy MSSM, leading to a lower upper bound on the lightest higgs mass. The discovery potential for the higgs boson is analyzed in section 6, and conclusions are presented in section 7.

## 2 Standard model vacuum stability bound

Recently it has been shown that when the newly reported value of the top mass is input into the effective potential for the SM higgs field, the broken-symmetry potential minimum is stable only if the SM higgs mass satisfies the lower bound constraint [7]:

$$m_H > 132 + 2.2(m_t - 170) - 4.5\left(\frac{\alpha_s - 0.117}{0.007}\right), \quad (1)$$

valid for a top mass in the range 160 to 190 GeV, and

$$m_H > 75 + 1.64(m_t - 140) - 3.0\left(\frac{\alpha_s - 0.117}{0.007}\right), \quad (2)$$

valid for a top mass in the range 130 to 150 GeV; for a top mass between 150 and 160 GeV, approximately 2 GeV must be added to the bound in Eq. (2). In these equations, mass units are in GeV, and  $\alpha_s$  is the strong coupling constant at the scale of the  $Z$  mass. These equations are the results of RGE-improved two-loop calculations, and include radiative corrections to the higgs and top masses. They are reliable, and accurate to 1 GeV in the top mass, and 2 GeV in the higgs mass [7].

If the universe is allowed to reside in an unstable minimum, then a similar, but slightly weaker (by  $\lesssim 5$  GeV for heavy  $m_t$  [7]) bound results. The unstable vacuum bound is only slightly weaker because the instability must be slight to preclude the possibility that early universe thermal fluctuations pushed the universe into the wrong but stable vacuum [8].

It has been known for some time [9] that the SM lower bound rises rapidly as the value of the top mass increases through  $M_Z$ ; below  $M_Z$  the bound is of order of the Linde-Weinberg value,  $\sim 7$  GeV [10]. So what is new here is the inference from the large reported value for  $m_t$  that the SM higgs lower mass bound dramatically exceeds 100 GeV! Adding the statistical and systematic errors of the CDF top mass measurement in quadrature gives a top mass with a single estimated error of  $m_t = 174 \pm 16$  GeV. The D0 collaboration has used its nonobservation of top candidates to report a 95% confidence level lower bound on the top mass of 131 GeV [11]. The D0 lower bound is predicated on the presumed dominance of the decay mode  $t \rightarrow b + W$ . The dominance of this mode is supported by the event signatures in the CDF data. We will assume the validity of the D0 lower mass bound <sup>3</sup>. Thus, the D0

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so any radiative correction to the SM higgs mass is not measurable. In contrast, in the MSSM the lightest higgs mass at tree-level is fixed by other observables, and so the finite renormalization is measurable.

<sup>3</sup>A top mass limit independent of the top decay modes is provided by an analysis of the  $W$  boson width:  $m_t > 62$  GeV at 95% confidence [12].

lower bound, and the CDF mass value including  $1\sigma$  allowances are, respectively, 131, 158, 174, and 190 GeV. Inputting these top mass values into Eqs. (1) and (2) with  $\alpha_s = 0.117$  then yields SM higgs mass lower bounds of 60, 106, 140, and 176 GeV, respectively <sup>4</sup>.

This lower limit on the SM higgs from the vacuum stability argument is a significant phenomenological constraint, and it rises linearly with  $m_t$ , for  $m_t \gtrsim 100$  GeV. On the other hand, the upper limit on the lightest MSSM higgs rises quadratically with  $m_t$ , also for  $m_t \gtrsim 100$  GeV. Thus, for very heavy  $m_t$ , the two bounds will inevitably overlap. Also, for relatively light  $m_t$  the bounds may overlap; e.g. we have just seen that the SM lower bound is 60 GeV for  $m_t = 131$  GeV, whereas for large or small  $\tan\beta$  the MSSM upper bound is at least the Z mass. However, for  $m_t$  heavy, but not too heavy, there may be no overlap. If so, the first measurement of the lightest higgs mass will serve to exclude either the SM higgs sector, or the MSSM higgs sector! In what follows, we show that in fact for  $m_t$  around the value reported by the CDF collaboration, there is a gap between the SM higgs mass lower bound and the MSSM upper bound.

The vacuum stability bound on the SM higgs mass is sensitive to the value of  $\alpha_s$ . We have taken  $\alpha_s = 0.117$  (the central value in the work of [7]) to produce the bounds displayed in Fig. 1. The reported LEP central value from event shape analyses is  $\alpha_s(M_Z) = 0.124 \pm 0.005$  [14]. Other LEP analyses, and deep inelastic leptonproduction data extrapolated to the  $M_Z$  scale give lower values, resulting in a world average of  $\alpha_s(M_Z) = 0.120 \pm 0.006 \pm 0.002$  [15]. If we use the generous value  $\alpha_s = 0.129$ , the lower bound on the SM higgs mass decreases by about 8 GeV for  $m_t > 160$  GeV. This will decrease the gap between the SM higgs lower bound and the MSSM higgs upper bound. However, a decrease of even this magnitude in the SM lower bound is compensated by the decrease in the MSSM upper bound due to two-loop contributions not included in our calculations, but discussed in section 3.

Since vacuum stability of the SM first breaks down for scalar field fluctuations on the order of  $10^6 - 10^{10}$  GeV [7], an implicit assumption in this SM bound is no new physics below  $10^{10}$  GeV. In particular, the stability bound, calculated with perturbation theory, is not valid if there is a non-perturbatively large value for the higgs self-coupling  $\lambda$  below  $\sim 10^{10}$  GeV. However, if there is a non-perturbatively large value for  $\lambda$  below  $10^{10}$  GeV, then there will be a Landau pole near or below  $10^{10}$  GeV, which in turn implies a *triviality lower bound* on the SM higgs mass of about 210 GeV. Before we show how this lower SM bound comes about, let us state the immediate consequence: *assuming no new fields with mass scales below  $10^{10}$  GeV, either the perturbative stability bound is valid for the SM higgs, or the non-perturbative triviality lower bound is valid.* The stability bound is the less restrictive, and we assume it in the subsequent sections of this paper.

We conclude this section by outlining the origin of this SM triviality lower bound,  $m_h \gtrsim 210$  GeV. Various perturbative [16] and non-perturbative [17, 18] studies have shown that a nontrivial (meaning non-vanishing low energy self-coupling) scalar model can only consistently be defined as a cut-off theory (i. e. an effective low energy theory that ignores new physics at and above the cut-off scale). An analysis of the one-loop renormalization group equation for the Higgs coupling already reveals some of the consequences of the non-

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<sup>4</sup>We learn here why the LEP experiments have established the non-existence of the SM higgs particle below a mass value of 64 GeV [13]: when fed into the vacuum stability argument, the heaviness of the top mass requires a low energy SM higgs desert!

asymptotically free running of the scalar coupling and its trivial (meaning zero, or nearly so) infra-red fixed point:

$$\mu \frac{d\lambda}{dt} = \frac{3}{2\pi^2} \lambda^2 \quad (3)$$

The solution to this equation can be written in the form

$$\lambda(\mu) = \frac{\lambda(\mu_0)}{1 - \frac{3}{2\pi^2} \ln(\frac{\mu}{\mu_0}) \lambda(\mu_0)} \quad (4)$$

It is clear that the coupling constant grows with increasing energy scale. Extending the solution beyond its perturbative range of validity, a pole, called a “Landau pole”, manifests itself when the denominator is equal to zero. The energy scale of the pole is therefore

$$\Lambda = \mu_0 \exp \frac{2\pi^2}{3} \frac{1}{\lambda(\mu_0)} \quad (5)$$

The occurrence of a Landau pole is usually interpreted as the onset of non-perturbative physics, or other new physics. A more rigorous treatment would replace Eq. (3) with two RG equations coupling the running of  $\lambda$  and the running of the top quark yukawa coupling. However, it is known that inclusion of the top quark terms only slightly alters the solution  $\lambda(\mu)$  and the position of the Landau pole [19].

A given value of the Higgs mass completely specifies the solution to the renormalization group equation. In particular, at  $\mu_0 = m_h$ , the value of the higgs mass

$$m_h^2 = 2v^2 \lambda(m_h) \quad (6)$$

as determined by the curvature of the effective potential at its minimum, fixes the boundary condition in Eq.(4) for  $\lambda(\mu)$ . Setting the arbitrary scale  $\mu_0$  to  $m_h$  in Eq.(5), and using Eq.(6) to eliminate  $\lambda(m_h)$ , one gets the one-to-one relation between the position  $\Lambda$  of the Landau pole and the higgs mass  $m_h$ :

$$m_h^2 \ln \left( \frac{\Lambda^2}{m_h^2} \right) = \frac{8}{3} \pi^2 v^2 \quad (7)$$

If the position of the Landau pole is known, then the higgs mass is determined implicitly by Eq. (7). If it is only known that the Landau pole is *above* a certain scale, say  $\bar{\Lambda}$ , then since the higgs mass falls with increasing  $\Lambda$ , the higgs mass is only known to be *below* the value  $m_h(\bar{\Lambda})$ ; this is the “triviality upper bound”. On the other hand, if it is only known that the Landau pole is *below* a certain scale  $\bar{\Lambda}$ , then since the higgs mass rises with decreasing  $\Lambda$ , the higgs mass is only known to be *above* the value  $m_h(\bar{\Lambda})$ . Thus, the assumption that the self-coupling becomes non-perturbative below a specific energy scale yields a minimum value for the Higgs mass, the “triviality lower bound”. The inverse of the assumption of no new physics below  $\sim 10^{10}$  GeV that underlies the vacuum stability perturbative lower bound on the higgs mass therefore implies a non-perturbative lower bound. From Eq.(7) we find that  $m_h(\Lambda = 10^{10} \text{ GeV}) = 210 \text{ GeV}$ . This qualitative discussion based on the perturbative renormalization group equation is corroborated by several non-perturbative studies, using the lattice [17] or Wilson renormalization flows [18], and remains valid even if yukawa and gauge couplings are included.

### 3 The lightest higgs in the MSSM

The spectrum of the higgs sector in the MSSM contains two CP-even neutral higgses,  $h$  and  $H$ , with  $m_h < m_H$  by convention, one CP-odd neutral higgs  $A$  and a pair of charged higgs  $H^\pm$ . A common convenience is to parameterize the higgs sector by the mass of the CP-odd higgs  $m_A$  and the vev ratio  $\tan\beta \equiv v_T/v_B$ . These two parameters completely specify the masses of the higgs particles at tree level

$$\begin{aligned} m_{H,h}^2 &= \frac{1}{2}(m_A^2 + m_Z^2) \pm \frac{1}{2}\sqrt{(m_A^2 - m_Z^2)^2 c_{2\beta}^2 + (m_A^2 + m_Z^2)^2 s_{2\beta}^2} \\ m_{H^\pm}^2 &= m_A^2 + m_W^2 \end{aligned} \quad (8)$$

implying for example that  $m_{H^\pm} > m_W$ , that the upper bound on the lightest higgs mass is given by

$$m_h \leq |\cos(2\beta)| M_Z, \quad (9)$$

that the lightest higgs mass vanishes at tree level if  $\tan\beta = 1$ , and that the masses  $m_H, m_A$ , and  $m_{H^\pm}$  all increase together as any one of them is increased. However, radiative corrections strongly modify the tree level predictions in the neutral [6, 20, 21, 22] and charged [23, 21, 24] higgs sectors. Some consequences are that the charged higgs can be lighter than the  $W$  gauge boson [24], that the  $\tan\beta = 1$  scenario, in which  $m_h = 0$  at tree level, is viable due to the possibility of a large radiatively generated mass [22], and that the upper bound on the lightest higgs mass is increased by terms proportional to  $m_t^4 \ln(m_{\tilde{t}}/m_t)$ , as advertised in the introduction <sup>5</sup> [6].

An important mechanism for the production of the neutral higgses in  $e^+e^-$  colliders is the brehmsstrahlung of a higgs by a  $Z$  gauge boson. Relative to the coupling of the SM higgs to two  $Z$  bosons, the  $ZZH$  coupling is  $\cos(\beta - \alpha)$  and the  $ZZh$  coupling is  $\sin(\beta - \alpha)$ , where  $\alpha$  is the mixing angle in the CP-even neutral higgs mass matrix. The angle is restricted to  $-\frac{\pi}{2} \leq \alpha \leq 0$ , and is given at tree level by

$$\tan 2\alpha = \frac{(m_A^2 + m_Z^2)}{(m_A^2 - m_Z^2)} \tan 2\beta. \quad (10)$$

From Eq.(??) it is seen that the limit  $m_A \rightarrow \infty$  is important for three reasons. First, it requires  $\alpha \rightarrow \beta - \pi/2$ , implying that  $\cos(\beta - \alpha) \rightarrow 0$ , i. e. , the heavy higgs decouples from the  $Z$  gauge boson. Secondly, it requires that  $\sin(\beta - \alpha) \rightarrow 1$ , i. e. , the light higgs behaves like the SM higgs. And thirdly,  $m_A \rightarrow \infty$  is the limit in which the tree level  $m_h$  saturates its maximal value given in Eq. (9) for any value of  $\tan\beta$ .

We calculate the one-loop corrected lightest MSSM higgs mass,  $m_h$  [26]. Included are the full one-loop corrections from the top/bottom quarks and squarks, and the leading-log corrections from the remaining fields (charginos, neutralinos, gauge bosons, and higgs

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<sup>5</sup> Note that in the susy limit,  $m_t = m_{\tilde{t}}$  and the fermion and boson loop contributions cancel each other. However, in the real world of broken susy,  $m_t \neq m_{\tilde{t}}$ , and the cancellation is incomplete. The top quark gets its mass from its yukawa coupling to the electroweak vev, whereas the scalar top mass arises from three sources, from D-terms, from the top yukawa coupling, but mainly from the insertion into the model of dimensionful soft susy-breaking parameters. The interplay of these diverse masses leads to the dramatic correction. Note that the correction grows logarithmically as  $m_{\tilde{t}}$  gets heavy, rather than decoupling! For heavy  $m_{\tilde{t}}$  the large logarithms can be summed to all orders in perturbation theory using renormalization group techniques. Interestingly, the effect is to lower the MSSM upper bound [25].

bosons). Recently, full one-loop corrections from all particles [27] have been calculated. Since the dominant corrections are due to the heavy quarks and squarks, full one-loop corrections from charginos, neutralinos, gauge and higgs bosons are well approximated by their leading logarithm terms used here. Two-loop corrections have recently been calculated also [28], for the limit  $\tan\beta \rightarrow \infty$ . Keeping only the leading  $m_t$  terms, these corrections have been extrapolated to all  $\tan\beta$ . The graphical result in ref. [28] shows a lowering of the MSSM upper bound by several GeV <sup>6</sup>. From this work [28], we estimate the gap to be wider by several GeV than the one-loop separation we show in Fig. 1. This widening further enables a higgs mass measurement to distinguish the SM and MSSM models.

We choose  $m_A$  and all squark mass parameters to be large, approximately 1 TeV <sup>7</sup>, in order to find the maximum light higgs mass. With respect to the squark mixing, we work in two extreme scenarios:

- (a) no mixing, i. e. ,  $\mu = A_t = A_b = 0$ , where  $\mu$  is the supersymmetric higgs mass parameter and  $A_i$ ,  $i = t, b$  are the trilinear soft supersymmetry breaking terms; and
- (b) maximal mixing with  $\mu = A_t = A_b = 1$  TeV.

The resulting lightest higgs mass as a function of  $\tan\beta$  is shown in Fig. 1 for the four experimentally motivated values of the top quark mass discussed earlier. For the case  $\tan\beta \sim 1$ , the SM lower bound and the MSSM upper bound are already non-overlapping at  $m_t = 131$  GeV. However, for larger  $\tan\beta$  values, the overlap persists until  $m_t \gtrsim 160$  GeV. For the preferred CDF value of  $m_t = 174$  GeV, the gap is present for all  $\tan\beta$ , allowing discrimination between the SM and the MSSM based on the lightest higgs mass alone. At  $m_t = 190$  GeV the gap is still widening, showing no signs of the eventual gap-closure at still higher  $m_t$ .

Also in Fig. 1 we see that scenario (b) offers a larger value for the  $m_h$  maximum than does scenario (a), except for the region  $\tan\beta \gg 1$ . The reason is that among the additional light higgs mass terms in (b) is a negative term proportional to  $-\mu^4 m_b^4 / c_\beta^4$ , which becomes large [25] when  $\tan\beta \gg 1$ . More significant is the fact that the extreme values in (a) and (b) yield a very similar upper bound in the region of acceptable  $\tan\beta$  values, thereby suggesting insensitivity of the MSSM upper bound to a considerable range of the squark mixing parameters.

It is known that the branching ratio  $B(b \rightarrow s\gamma)$  has a strong dependence on the susy higgs parameters [33, 34]. However, when all squarks are heavy, as here, the contribution from the chargino/squark loops to  $B(b \rightarrow s\gamma)$  is suppressed. In the case of heavy squarks, the charged-higgs/top-quark loop may seriously alter the rate, and strong constraints on the charged higgs minimum mass result [35, 34]. This constraint does not affect the present work, where we take  $m_A$  and therefore  $m_{H^\pm}$  and  $m_H$  large in order to establish the light higgs upper bound: in the large  $m_A$ , large squark mass limit, the ratio  $B(b \rightarrow s\gamma)$  approaches the SM value, consistent with the CLEO bound [36].

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<sup>6</sup>In ref. [29] were found small and positive two-loop contributions of the order  $m_t^6$ ; however, the QCD two-loop contributions found in ref. [28] are of order  $\alpha_s^2 m_t^4$ , are negative, and dominate the previous ones. The net effect is to lower the higgs mass bound.

<sup>7</sup>We note that  $\lesssim 1$  TeV emerges naturally for the heavier superparticle masses when the MSSM is embedded into a GUT [30, 31, 32].

## 4 The lightest higgs in non-standard susy models

The MSSM can be extended in a straightforward fashion by adding an  $SU(2)$  singlet  $S$  with vanishing hypercharge to the theory [37]. As a consequence, the particle spectrum contains an additional scalar, pseudoscalar, and neutralino. This extended model, the so-called (M+1)SSM, features four possible additional terms in the superpotential. Two of these terms,  $\lambda SH_B \epsilon H_T$  and  $\frac{1}{3}\kappa S^3$ , enter into the calculation of the lightest higgs mass;  $\epsilon$  is the usual antisymmetric 2 by 2 matrix.

A tree-level analysis of the eigenvalues of the scalar mass matrix yields an upper bound on the mass of the lightest higgs boson:

$$m_h^2 \leq M_Z^2 \left\{ \cos^2 2\beta + 2 \frac{\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right\}. \quad (11)$$

The first term on the right hand side is just the MSSM result of Eq. (9). The second term is positive semidefinite, and so weakens the bound compared to its counterpart in the MSSM. Moreover, the parameter  $\lambda$  is *a priori* free, and so the second term may considerably weaken the upper bound [38, 39, 40]. However, there are two cases where the bound will suffer only a minor adjustment. The first is the large  $\tan \beta$  scenario, where  $\cos^2 2\beta$  is necessarily  $\gg \sin^2 2\beta$ . The second is when the theory is embedded into a GUT. In this case, the strength of  $\lambda$  at the susy-breaking scale,  $M_{SUSY}$ , is limited: even if  $\lambda$  assumes a high value at the GUT scale, the nature of the renormalization group equations is such that its evolved value at the susy-breaking scale is a rather low, pseudo-fixed point. Under the assumption that all coupling constants remain perturbative up to the GUT scale, it is therefore possible to calculate a maximum value for the mass of the lightest higgs boson [38, 39]. It turns out that this lightest mass upper bound occurs when  $\kappa$  is close to zero. The higgs mass upper bound depends on the value of the top yukawa  $g_t$  at the GUT scale through the renormalization group equations. Above  $M_{SUSY}$  the running of the coupling constants is described by the (M+1)SSM renormalization group equations, whereas below this scale the SM renormalization group equations are valid. At  $M_{SUSY}$  the boundary conditions

$$\begin{aligned} \lambda^{SM} &= \frac{1}{8} (g_1^2 + g_2^2) \left( \cos^2 2\beta + 2 \frac{\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right), \\ g_t^{SM} &= g_t \sin \beta, \end{aligned} \quad (12)$$

incorporate the transition from the (M+1)SSM to the SM. Here  $\lambda^{SM}$  and  $g_t^{SM}$  are the standard model higgs self coupling and top quark yukawa coupling respectively. The value of the higgs boson mass is determined implicitly by the equation  $2\lambda^{SM}(m_h)v_{SM}^2 = m_h^2$ . This RGE procedure of running couplings from  $M_{SUSY}$  down takes into account logarithmic radiative corrections to the higgs boson mass, in particular those caused by the heavy top quark.

In Fig. 2 we show the maximum value of the higgs boson mass as a function of  $\tan \beta$  for the chosen values of the top quark mass  $m_t$ . We have adopted a susy-breaking scale of  $M_{SUSY} = 1 \text{ TeV}$ ; this value is consistent with the notion of stabilizing the weak-to-susy GUT hierarchy, and is the value favored by RGE analyses of the observables  $\sin^2 \theta_W$  and  $m_b/m_\tau$ . The bounds in Fig. 2 are quite insensitive to the choice of  $M_{SUSY}$ , increasing very



slowly as  $M_{SUSY}$  increases [38]. We have assumed that all superpartners and all higgs bosons except for the lightest one are heavy, i. e.  $\sim M_{SUSY}$ . In Fig. 2 it is revealed that for low values of the top quark mass ( $\sim M_Z$ ), the mass upper bound on the higgs boson in the (M+1)SSM will be substantially higher than in the MSSM at  $\tan\beta \lesssim$  a few. This is because  $\lambda(m_h)$  is large for low  $m_t$ , and because  $\sin^2 2\beta \gtrsim \cos^2 2\beta$  for  $\tan\beta \lesssim$  a few. However, for a larger top quark mass the difference between the MSSM and (M+1)SSM upper bounds diminishes. This is because  $\lambda(m_h)$  falls with increasing  $m_t$ , and because there is an increasing minimum value for  $\sin\beta = g_t^{SM}/g_t$  [from the second of Eqs. (12)], and therefore for  $\tan\beta$ , when  $m_t \propto g_t^{SM}$  is raised and  $g_t$  is held to be perturbatively small up to the GUT scale. This increasing minimum value of  $\tan\beta$  is evident in the curves of Fig. 2. A comparison of Figs. 1 and 2 reveals that the (M+1)SSM and MSSM bounds are very similar at  $\tan\beta \gtrsim 6$ . For  $m_t$  at or above the CDF value, only this  $\tan\beta \gtrsim 6$  region is viable in the (M+1)SSM model. Since the (M+1)SSM model was originally constructed to test the robustness of the MSSM, it is gratifying that the two models show a very similar upper bound.

The results for more complicated extensions of the minimal model tend to be similar [40]. In general, the mass of the lightest higgs boson at tree level is limited by  $M_Z$  times a factor proportional to the dimensionless coupling constants in the higgs sector. The requirement of perturbative unification restricts the value of these coupling constants at the electroweak scale, and the maximum value of the lightest higgs boson mass is therefore never much larger than  $M_Z$ .

We have seen that the SM, MSSM, and the (M+1)SSM electroweak models can be disfavored or ruled out by a measurement of  $m_h$ ; and that a “forbidden” mass gap exists for  $m_t \gtrsim 160$  GeV. We next give an example of a non-standard susy model that cannot be embedded in a GUT, and requires a low susy breaking scale: a gauged, non-linear, supersymmetric sigma model. The simplest supersymmetric model with a non-linear representation of the  $SU(2) \times U(1)$  symmetry is obtained by imposing the constraint  $H_T \epsilon H_B = \frac{1}{4} v_{SM}^2 \sin^2 2\beta$  on the action of the MSSM [41]. This constraint is the only one possible in the MSSM higgs sector that obeys supersymmetry, is invariant under  $SU(2) \times U(1)$ , and leaves the vev in a global minimum<sup>8</sup>. As a result of this constraint one of the scalar higgs bosons, the pseudoscalar, and one of the neutralinos are eliminated from the particle spectrum. The remaining higgs boson has a mass  $m_h^2 = M_Z^2 + (\hat{m}_T^2 + \hat{m}_B^2) \sin^2 2\beta$ , and the charged higgs bosons have masses  $m_{H^\pm}^2 = M_W^2 + (\hat{m}_T^2 + \hat{m}_B^2)$ . Here,  $\hat{m}_T^2$  and  $\hat{m}_B^2$  are soft, dimensionful, susy-breaking terms; they may be positive or negative.

In order for the notion of a supersymmetric non-linear model to be relevant, the susy breaking scale is required to be much smaller than the chiral symmetry breaking scale  $4\pi v^{SM}$ . The natural magnitude for the parameters  $\hat{m}_B^2$  and  $\hat{m}_T^2$  is therefore of the order of  $M_Z^2$ . Consequently, both the neutral and the charged higgs bosons have masses of at most a few multiples of  $M_Z$  in the non-linear model. This formalism of the effective action allows a description of the low energy physics independent of the particular strongly-interacting underlying theory from which it derives. Thus we believe that the non-linear MSSM model

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<sup>8</sup>This MSSM non-linear sigma model is not the formal heavy higgs limit of the MSSM. This is in contrast to the non-linear sigma models that result from the heavy higgs limit of the SM, or of the (M+1)SSM. The difference is that MSSM does not contain an independent, dimensionless, quartic coupling constant  $\lambda$  in the higgs sector which can be taken to infinity, whereas the SM and (M+1)SSM do.

presented here is probably representative of a class of underlying strongly-interacting susy models. The lesson learned then is that measuring a value for  $m_h$  at  $\lesssim 300$  GeV cannot validate the SM, MSSM, (M+1)SSM, or any other electroweak model. However, the premise of this present article remains valid, that such a measurement should rule out one or more of these popular models.

## 5 Supersymmetric Grand Unified Theories

Supersymmetric grand-unified theories (susy GUTs) are the only simple models in which

- (i) the three low energy gauge coupling constants are known to merge at the GUT scale;
- (ii) the correct low energy value for the weak mixing angle  $\sin \theta_W$  is obtained;
- (iii) hierarchy and parameter-naturalness issues are solved.

Thus, it is well motivated to consider the grand unification of the low energy susy models. Many susy GUTs reduce at low energies to the MSSM with additional constraints on the parameters [31]. The additional constraints must yield an effective low energy theory that is a special case of the general MSSM we have just considered. Accordingly, the upper limit <sup>9</sup> on  $m_h$  in such susy GUTs is in general more restrictive than the bound presented in section 3. The assumption of gauge coupling constant unification (with its implied desert between  $M_{SUSY}$  and  $M_{GUT}$ ) presents no significant constraints on the low energy MSSM parameters [31, 42]. However, the further assumption that the top yukawa coupling remains perturbatively small up to  $M_{GUT}$  leads to the low energy constraint  $0.96 \leq \tan \beta$ . This is because the RGE evolves a large but perturbative top yukawa coupling at  $M_{GUT}$  down to its well-known infrared pseudo-fixed-point value at  $M_{SUSY}$  and below, resulting in the top mass value  $\sim 200 \sin \beta$  GeV. If the bottom yukawa is also required to remain perturbatively small up to  $M_{GUT}$ , then  $\tan \beta \leq 52$  [43] emerges as a second low energy constraint.

The pseudo-fixed-point solution is not a true fixed-point, but rather is the low energy yukawa value that runs to become a Landau pole (an extrapolated singularity, presumably tamed by new physics) near the GUT scale. The apparent CDF top mass value is within the estimated range of the pseudo-fixed-point value. Thus it is attractive to assume the pseudo-fixed-point solution. With the additional assumptions that the electroweak symmetry is radiatively broken [44] (for which the magnitude of the top mass is crucial) and that the low energy MSSM spectrum is defined by a small number of parameters at the GUT scale (the susy higgs mass parameter  $\mu$  – which is also the higgsino mass, and four universal soft susy-breaking mass parameters: the scalar mass, the bilinear and trilinear masses, and the gaugino mass), two compact, disparate ranges for  $\tan \beta$  emerge:  $1.0 \leq \tan \beta \leq 1.4$  [43], and a large  $\tan \beta$  solution  $\sim m_t/m_b$ . Reference to our Figs. 1 and 2 shows that the gap between the SM and MSSM is maximized in the small  $\tan \beta$  region and minimized in the large  $\tan \beta$  region, whereas just the opposite is true for the SM and (M+1)SSM models. Moreover, the (M+1)SSM model is an inconsistent theory in the small  $\tan \beta$  region if  $m_t \gtrsim 160$  GeV.

In fact, a highly constrained low  $\tan \beta$  region  $\sim 1$  and high  $\tan \beta$  region  $\gtrsim 40$ –70 also emerge when bottom- $\tau$  yukawa unification at the GUT scale is imposed on the radiatively

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<sup>9</sup>In fact, the additional restrictions may be so constraining as to also yield a *lower* limit on the lightest higgs mass, in addition to the upper limit. For example,  $m_h > 85$  GeV for  $\tan \beta > 5$  and  $m_t = 170$  GeV is reported in ref.[31], and a similar result is given in [32].

broken model [45, 46, 47, 48, 49, 50]. Bottom- $\tau$  yukawa coupling unification is attractive in that it is natural in susy SU(5), SO(10), and  $E_6$ , and explains the low energy relation,  $m_b \sim 3m_\tau$ . With bottom- $\tau$  unification, the low to moderate  $\tan\beta$  region requires the proximity of the top mass to its fixed-point value [51], while the high  $\tan\beta$  region also requires the proximity of the bottom and  $\tau$  yukawas to their fixed-point; the emergence of the two  $\tan\beta$  regions results from these two possible ways of assigning fixed-points.

The net effect of the yukawa-unification constraint in susy GUTs is necessarily to widen the mass gap between the light higgs MSSM and the heavier higgs SM, thus strengthening the potential for experiment to distinguish the models. The large  $\tan\beta$  region is disfavored by proton stability [52]. Adoption of the favored low to moderate  $\tan\beta$  region leads to a highly predictive framework for the higgs and susy particle spectrum [48, 49, 50]. In particular, the fixed-point relation  $\sin\beta \sim m_t/(200\text{GeV})$  fixes  $\tan\beta$  as a function of  $m_t$ . For a heavy top mass as reported by CDF, one has  $\tan\beta \sim (1, 2)$  for  $m_t = (140, 180)$  GeV. Since  $\tan\beta \sim 1$  is the value for which the  $m_h$  upper bound is minimized (the tree-level contribution to  $m_h$  vanishes), the top yukawa fixed-point models offer a high likelihood for  $h^0$  detection at LEP200. Reduced  $m_h$  upper bounds have been reported in [46, 47, 48, 50]. These bounds are basically our bound in Fig. 1 for  $\tan\beta \gtrsim 1$ , where small differences appear when different methods and approximations are used. These reduced bounds are due to the small  $\tan\beta$  restriction, an inevitable consequence of assigning the top mass, but not the bottom mass, to the pseudo fixed-point.

Even more restrictive susy GUTs have been analyzed. These include the “no-scale” or minimal supergravity models [53], in which the soft mass parameters  $m_0$  (universal scalar mass) and  $A$  are zero at the GUT scale; and its near relative, the superstring GUT, in which the dilaton vev provides the dominant source of susy breaking and so  $m_0$ ,  $A$ , and the gaugino mass parameter all scale together at the GUT scale [54]. Each additional constraint serves to further widen the SM/MSSM higgs mass gap.

In radiatively broken susy GUTs with universal soft parameters, the superparticle spectrum emerges at  $\lesssim 1$  TeV. If the spectrum in fact saturates the 1 TeV value, then as we have seen the Feynman rules connecting  $h^0$  to SM particles are indistinguishable from the Feynman rules of the SM higgs. Thus, it appears that if a susy GUT is the choice of Nature, then the mass of the lightest higgs, but not the higgs production or dominant decay modes, may provide our first hint of grand unification.

## 6 Discovery potential for the higgs boson

The higgs discovery potential of LEP2 [55, 56] depends on the energy at which the machine is run. A higgs mass up to 105 GeV is detectable at LEP2 with the  $\sqrt{s} = 200$  GeV option (LEP200), while a higgs mass only up to 80 GeV is detectable with LEP178. As we have shown, the large value of  $m_t$  reported by CDF raises the upper limit on the MSSM  $h^0$  mass to  $\sim 130$  GeV. Near this limit the MSSM higgs has the production and decay properties of the SM higgs. Discovery of this lightest MSSM higgs then argues strongly for the LEP200 option over LEP178. Furthermore, for any choices of the MSSM parameters, associated production of either  $h^0 Z$  or  $h^0 A$  is guaranteed at LEP200 as long as  $m_{\tilde{t}} \lesssim 300$  GeV [55]. Even better would be LEP230, where detection of  $Zh^0$  is guaranteed as long as  $m_{\tilde{t}} \lesssim 1$  TeV

[55]. At an NLC300 (the Next Linear Collider), detection of  $Zh^0$  is guaranteed for MSSM or for (M+1)SSM [55]. Turning to hadron colliders [57, 58], it is now believed that while the higgs cannot be discovered at Fermilab's Tevatron with its present energy and luminosity, the mass range 80 GeV to 130 GeV is detectable at any hadron collider with  $\sqrt{s} \gtrsim 2$  TeV and an integrated luminosity  $\mathcal{L} \gtrsim 10\text{fb}^{-1}$  [58]; the observable mass window widens significantly with increasing luminosity, but very little with increasing energy. For brevity, we will refer to this High Luminosity DiTevatron hadron machine as the "HLDT". If the SM desert ends not too far above the electroweak scale, then the SM higgs may be as heavy as  $\sim 600\text{--}800$  GeV <sup>10</sup> (but not heavier, according to the triviality argument), in which case only the LHC (and not even NLC500) guarantees detection.

We present our conclusions on detectability for each of the four  $m_t$  values that we have considered <sup>11</sup> :

(i) if  $m_t \sim 131$  GeV, then the SM higgs mass lower bound from vacuum stability is 60 GeV; a SM mass up to (80,105,130) GeV is detectable at (LEP178,LEP200,HLDT); and the MSSM  $h^0$  is certainly detectable at LEP178 for  $\tan\beta \sim 1\text{--}2$ , and certainly detectable at LEP200 for all  $\tan\beta$ .

(ii) if  $m_t \sim 158$  GeV, then the SM lower bound rises above 100 GeV, so the SM higgs cannot be detected at LEP178 or LEP200, but is still detectable at the HLDT if its mass is below 130 GeV; the lightest MSSM higgs is certainly detectable at LEP178 if  $\tan\beta$  is very close to 1, and is certainly detectable at LEP200 if  $\tan\beta$  is  $\lesssim 3$ .

(iii) if  $m_t \sim 174$  GeV, then the SM higgs is above 140 GeV, out of reach for LEP200 and the HLDT; the MSSM higgs is certainly detectable at LEP200 if  $\tan\beta \sim 1\text{--}2$ .

(iv) if  $m_t \sim 190$  GeV, then the SM higgs is above 176 GeV in mass; at any  $\tan\beta$  value, the MSSM higgs is not guaranteed to be detectable at LEP200, but is certainly detectable at the HLDT if  $\tan\beta \sim 1\text{--}3$ .

It is interesting that the  $h^0$  mass range is most accessible to experiment if  $\tan\beta \sim 1\text{--}3$ , just the parameter range favored by susy GUTs.

## 7 Discussion and conclusions

We repeat that the lightest MSSM higgs is guaranteed detectable at LEP230; and that the lightest (M+1)SSM higgs and MSSM higgs are guaranteed detectable at a NLC300 and at the LHC. Since there is no lower bound on the lightest MSSM higgs mass other than the experimental bound, the MSSM  $h^0$  is possibly detectable even at LEP178 for all  $\tan\beta$ , but there is no guarantee. The SM higgs is guaranteed detectable only at the LHC; if  $m_t \sim 174$  GeV, then the SM higgs will not be produced until the LHC or NLC is available. Thus, one simple conclusion is that LEP200 has a tremendous potential to distinguish MSSM and (M+1)SSM symmetry breaking from SM symmetry breaking.

It is worth noting that with enough higgs events, measurement of certain rare decay modes is very sensitive to non-SM higgs physics. For example, modes forbidden at tree-level but induced at one loop, such as  $h \rightarrow \gamma\gamma$ ,  $h \rightarrow \gamma Z$ , and  $Z \rightarrow \gamma h$ , receive comparable

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<sup>10</sup>Theorists would prefer an even lower value of  $\lesssim 400$  GeV, so that perturbative calculations in the SM converge [59].

<sup>11</sup>Recall that the SM vacuum stability bound assumes a desert up to at least  $10^{10}$  GeV

contributions from standard and superpartner particles in the loop. The branching fractions for these modes may vary by an order of magnitude or more from the SM values [60]. However, measurements of these rare modes will require the LHC or the photon-photon collider option of the NLC500 [58].

Thus, either the direct detection of the lightest higgs particle as discussed herein or measurements of rare higgs decays have the potential to distinguish the SM and MSSM symmetry breaking sectors. The mass measurement will come first. We have shown that for a top quark mass  $\sim 174$  GeV, as reported by CDF, a gap exists between the SM higgs mass ( $\gtrsim 140$  GeV) and the lightest MSSM higgs mass ( $\lesssim 130$  GeV). Thus, the first higgs mass measurement will eliminate one of these popular models. Most of the MSSM mass range is accessible to LEP II. If a higgs is discovered at LEP II, the SM higgs sector is ruled out. For the (M+1)SSM with the assumption of perturbative unification, conclusions remain the same as for the MSSM.

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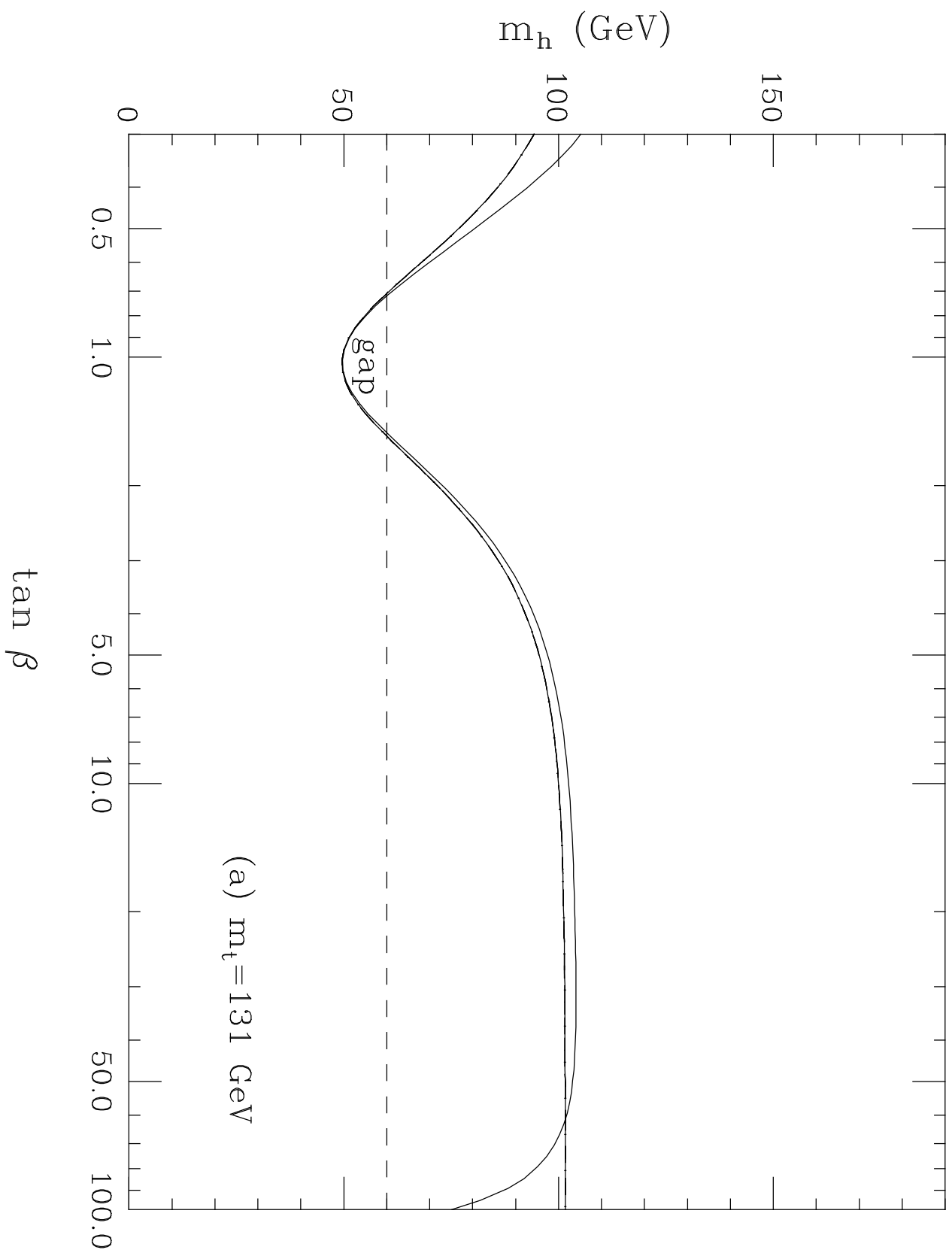
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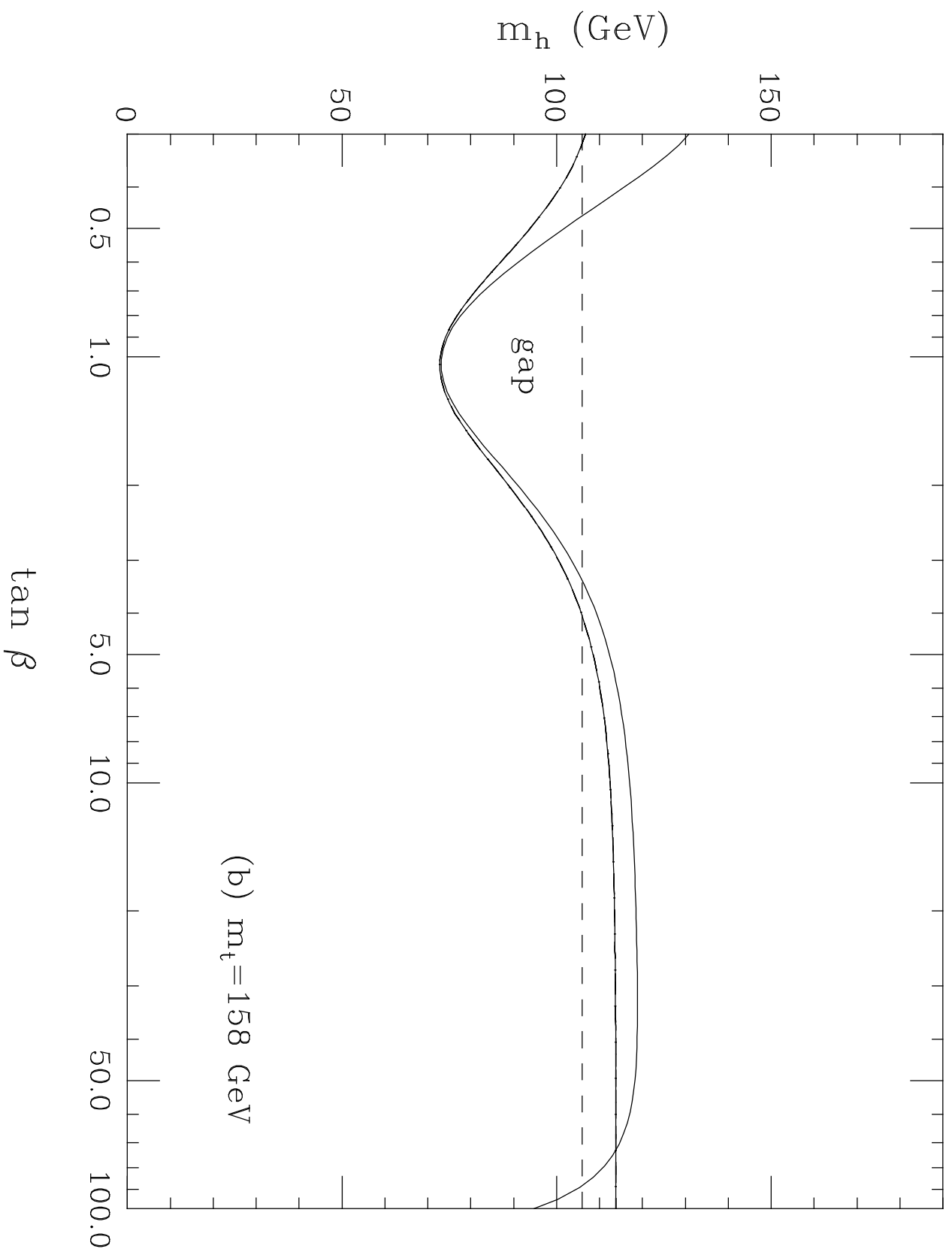
**Fig. 1** The solid curves reveal the upper bound on the lightest MSSM higgs particle vs.  $\tan\beta$ , for top mass values of (a) 131 GeV, (b) 158 GeV, (c) 174 GeV, and (d) 190 GeV. Two extreme choices of susy parameters are invoked: the higher curve is for  $\mu = A_t = A_b = 1$  TeV, and the lower curve is for  $\mu = A_t = A_b = 0$ ; in both cases,  $m_A = m_{\tilde{q}} = 1$  TeV and  $m_b(M_Z) = 4$  GeV are assumed. The dashed curve is the ( $\tan\beta$  independent) lower bound on the SM higgs mass such that the universe sits in the SM vacuum since the time of the electroweak phase transition.

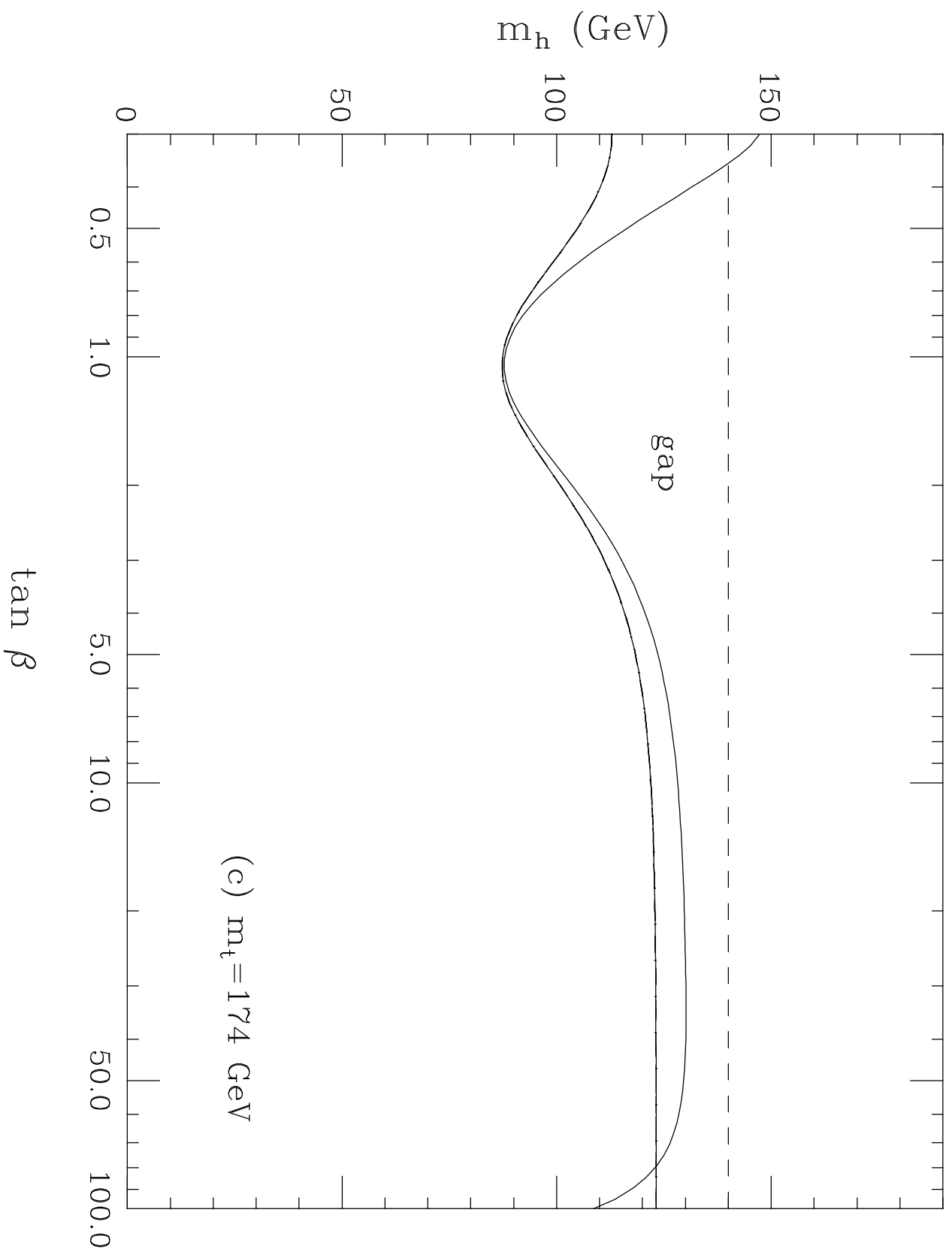
**Fig. 2** Upper bound on the lightest (M+1)SSM higgs vs.  $\tan\beta$ , for the top mass values (a) 131 GeV, (b) 158 GeV, (c) 174 GeV, and (d) 190 GeV. All superparticles and higgses beyond the lightest are assumed to be heavy, at the chosen susy-breaking scale of 1 TeV. The GUT scale is taken as  $10^{16}$  GeV.

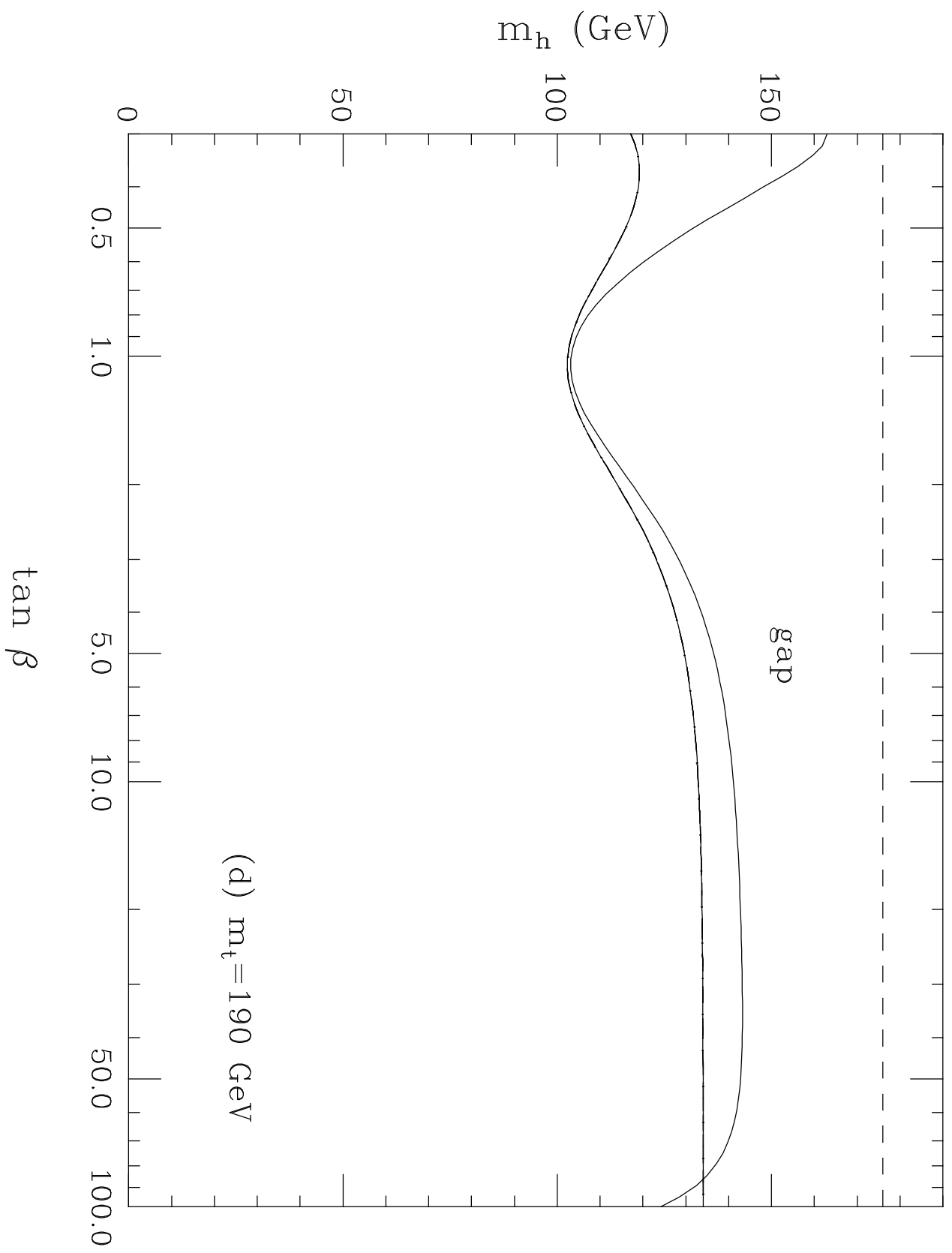
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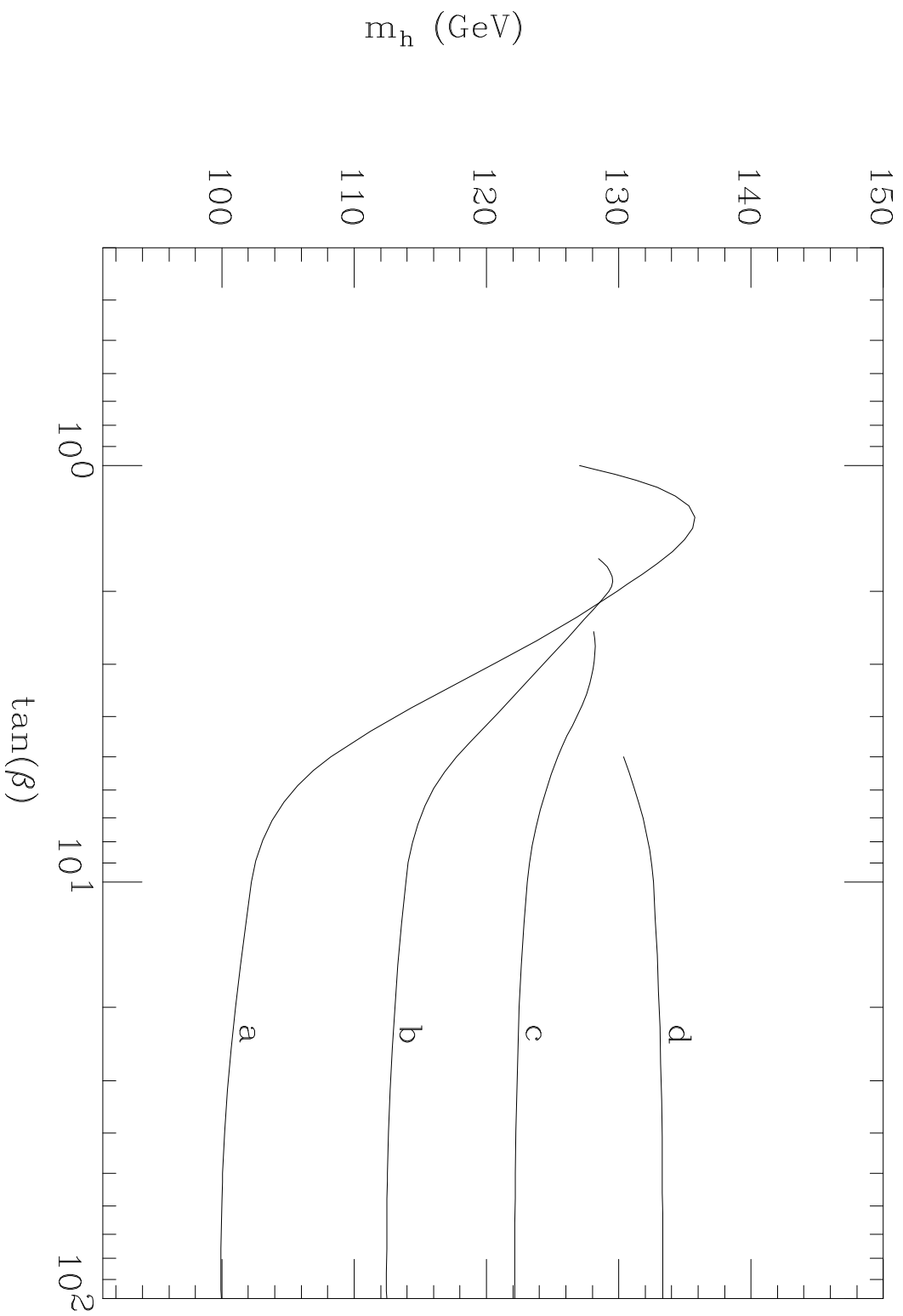


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Figure 2



This figure "fig1-3.png" is available in "png" format from:

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